Factor Timing: Dynamic Portfolio Allocation via Machine Learning

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Abstract

This paper explores the application of machine learning, specifically a multinomial logistic regression model, for dynamic factor timing in asset allocation. By incorporating a diverse set of macroeconomic, sentiment, momentum, and valuation indicators, the model aims to predict the relative performance of investment factors, including Size, Value, Momentum, Quality, and Low Volatility. The study employs backward selection to identify the most significant predictors, resulting in a model that enhances portfolio performance compared to traditional static allocation methods. Out-of-sample testing shows that the model-based portfolios, particularly the Hierarchical Portfolio, achieve higher risk-adjusted returns than equal-weighted and Sharpe-optimized portfolios. While the results demonstrate the potential of machine learning in optimizing factor allocation, the gap between the model's performance and that of an idealized Best Portfolio suggests room for further refinement and innovation in this field.

1. Introduction

Factor investing has experienced significant growth in recent years as reported by ETF.com (2024). This surge has brought forth new challenges in asset allocation, particularly in the context of effectively timing factor exposures to optimize returns. Traditional asset allocation methods, such as mean-variance optimization (Markowitz, 1967), Sharpe Ratio optimization (Sharpe, 1966) and the Black-Litterman model (Black & Litterman, 1992), have long been the cornerstone of portfolio management. These approaches, including equal-weighting and various optimization strategies, have provided foundational frameworks for balancing risk and return. Moreover, studies suggest that imposing constraints on portfolio weights can enhance performance by mitigating estimation risk and reducing sensitivity to input errors (Frost & Savarino, 1988).

Factor timing, the practice of adjusting exposure to various investment factors based on prevailing market conditions, has proven to be a challenging endeavor, often limited by the complexities of accurately predicting factor returns based on factor fundamentals (Asness et al., 2017; S. Asness, A. Friedman, J. Krail, & M. Liew, 2000). Other approaches frequently rely on regression models that attempt to forecast the excess returns of different assets, subsequently allocating resources to those predicted to yield the highest returns (Pinelis & Ruppert, 2022).

This paper contributes to the evolving literature by proposing a novel approach to factor timing through the application of machine learning models. Unlike conventional methods that focus on predicting absolute returns, this study employs machine learning techniques to forecast the relative performance of different factors. By incorporating a diverse set of exogenous variables, this approach aims to create a more robust and flexible framework capable of adapting to varying market conditions.

Building on the foundational work of Black and Litterman (1990, 1992), Sharpe (1964), and Markowitz (1967), this research integrates contemporary advancements in factor investing (Fama & French, 2012, 2015 ; Frazzini, Kabiller, & Pedersen, 2018) and machine learning (Nazaire, Pacurar, & Sy, 2021 ; Ma, Yang, Luo, Li, & He, 2024 ; Sengupta, Jana, Dutta, & Mukherjee, 2024). The study rigorously compares the proposed machine learning-based model against traditional asset allocation methods to evaluate its potential in redefining the standards of asset management and factor timing. By doing so, this paper aims to demonstrate the transformative potential of machine learning in enhancing portfolio performance and addressing the challenges associated with modern asset allocation strategies.

2. Literature Review

With the advent of modern computational power, the intersection of machine learning and finance has gained significant traction, leading to a surge of research exploring these advanced tools in portfolio allocation. Studies have provided substantial economic and statistical evidence supporting the use of machine learning techniques in optimizing portfolio decisions (Pinelis & Ruppert, 2022). However, a notable challenge associated with many machine learning algorithms, such as neural networks, is their 'black box' nature, which obscures the interpretability of results. This lack of transparency can hinder the practical application of these models, as it diminishes the ability to understand the importance or correlation of predictors with the target variable, leading to potential skepticism about the robustness of the models (Koratamaddi, Wadhwani, Gupta, & Sanjeevi, 2021).

To address this issue, alternative algorithms like XGBoost and Random Forest have been employed, as they offer the advantage of visualizing feature importance while still delivering strong predictive performance (Caparrini, Arroyo, & Escayola Mansilla, 2024). These models strike a balance between accuracy and interpretability, making them more suitable for practical application where human judgment and confidence in the model's robustness are crucial.

Effective asset allocation goes beyond merely selecting assets with the highest expected returns; it requires a nuanced approach that considers the unique characteristics of each asset within a portfolio, often necessitating weight restrictions to comply with securities trading regulations (Ma, Yang, Luo, Li, & He, 2024). Additionally, accounting for volatility clustering has been shown to reduce potential drawdowns, thereby enhancing the resilience of portfolios (Simonato & Denault, 2023). This highlights the importance of considering economic market conditions, which can influence company returns and, consequently, the performance of various factors across different economic cycles. The integration of macroeconomic variables into portfolio allocation models is therefore a common and effective practice (Pinelis & Ruppert, 2022).

The foundations of factor investing can be traced back to seminal works in the field. Banz (1981) first identified the size factor, demonstrating that smaller firms tend to outperform larger ones. The value factor was later discovered by De Bondt and Thaler (1985), Lakonishok, Shleifer, and Vishny (1994), and Fama and French (1992), leading to the development of the Fama-French three-factor model, which includes the risk-free asset, the size factor, and the value versus growth factor (Fama & French, 1993). Building on this framework, Carhart (1997) introduced the four-factor model, incorporating the momentum factor, which showed that stocks with positive momentum tend to continue delivering strong returns, while those with negative momentum tend to underperform.

Further advancements include the identification of the ‘betting against beta’ factor by Frazzini and Pedersen (2014), which posits that assets with higher beta typically have lower alpha, challenging conventional risk-return assumptions. More recently, Asness, Frazzini, and Pedersen (2019) proposed the ‘quality-minus-junk’ factor, arguing that investors should place a premium on attributes such as profitability, growth, and safety, as these characteristics are associated with higher risk-adjusted returns. Research has also shown that diversification strategies based on smart-beta can effectively manage portfolio risk, reinforcing the value of factor-based approaches in modern asset management (Nazaire, Pacurar, & Sy, 2021).

3. Methodology

Variables

In this study, we construct a portfolio comprising five key factors: Size, Value, Momentum, Quality, and Low Volatility, as outlined in the literature review. To ensure the reproducibility of our results, we utilize synthetic indices provided by MSCI, which represent these factors within the U.S. market. The data spans from January 1, 2004, to July 9, 2024, providing a robust time series of over 20 years for our analysis. Unlike traditional methods that attempt to estimate the returns of each factor individually, we adopt a novel approach by creating a dummy variable, referred to as the 'Forward Factor'. This variable identifies the factor that achieves the highest relative return in the subsequent month, thereby transforming our task into a classification problem rather than a regression problem. This approach not only simplifies the predictive process but also aligns more closely with the practical goal of optimizing factor allocation based on expected performance.

To develop the predictive model, we collected 14 different exogenous variables with the goal of identifying the optimal configuration for the final model. To achieve this, we will employ a method known as "backward selection", which iteratively eliminates variables to minimize the Akaike Information Criterion (AIC).

$$AIC = 2k - 2\ln(\hat{L}) \quad (1)$$

In equation (1), k represents the number of estimated parameters, and $\hat{L}$ represents the maximum value of the model's likelihood function. Given the diverse nature of the variables, they have been categorized into four groups: Momentum, Valuation, Sentiment, and Macro. Below, a detailed description of each category and the specific variables is included, along with their relevance to the model.

Momentum Variables

Momentum indicators, as shown in Figure 3, are critical for detecting trends within factors. Short-term correlations, such as the relationship between Low Volatility and Quality or Low Size and Momentum, can provide actionable insights for rebalancing decisions. The RSI aids in identifying overbought or oversold conditions, which could signal opportunities for the upcoming rebalance. The Ratio of S&P 500 companies trading above their 200-day moving average variable offers an objective measure of the broader market trend.

- 1-Month Factor Return: Monthly return for each factor used.
- 12-Month Factor Return: Return over the last 12 months, excluding the most recent month, for each factor.
- RSI (Relative Strength Index): Average RSI of S&P 500 companies over the last 20 days.
- Above SMA: Ratio of S&P 500 companies trading above their 200-day moving average.

Valuation Variables

The Risk Premium is a key indicator of market stress, making it useful for determining when to shift the portfolio towards defensive factors like Low Volatility or Quality. Similarly, the long-term earnings growth (Gordon, 1962) helps in identifying undervaluations. Given the strong negative correlation between these two variables, it is likely that the backward selection process will retain only one of them. Both indicators are presented in Figure 5.

- LT Earnings Growth: Long-term earnings growth rate, calculated as $Earnings\ Yield\ (\frac{E}{P}) + g$, where g represents expected earnings growth in perpetuity, following the Gordon (1962) growth model.
- Risk Premium: The difference between the current estimated earnings yield and the yield on the 10-year Treasury note.

Sentiment Variables

The VIX typically remains at stable levels, with sharp increases during periods of economic stress or crises. A higher VIX is often associated with better performance by the Low Volatility factor. Conversely, short interest tends to exhibit lower volatility but could still provide useful signals, particularly for the Momentum factor. The time series of the variables are displayed in Figure 4.

- SI Ratio: Average short interest of S&P 500 companies.
- VIX (Volatility Index): A measure of market volatility.

Macro Variables

These macroeconomic indicators, as shown in Figure 2, offer a comprehensive view of the U.S. economy, encompassing aspects such as inflation, interest rates, and economic activity. They are likely to correlate strongly with factors like Quality and Low Volatility, providing essential context for portfolio adjustments.

- NFCI: Chicago Fed National Financial Conditions Index.
- Spread Yield: US High Yield Index Option-Adjusted Spread.
- Break-even Infl.: 10-Year Breakeven Inflation Rate.
- New Order: Manufacturers' New Orders for Durable Goods.
- CPI: Consumer Price Index for All Urban Consumers.
- ShortM Yield: Market Yield on U.S. Treasury Securities at a 3-Month Constant Maturity.

Model and Portfolio

Building on the predictors and target variable discussed in the previous sections, this study proposes the use of a multinomial logistic regression model, the formula for which is shown in equation (2). The target variable in this model is a categorical dummy variable representing the factor that is expected to have the highest return in the subsequent month.

$$P(Y_i = k) = \frac{exp\left(\beta_{k0} + \beta_{k1}X_1 + \beta_{k2}X_2 + \cdots + \beta_{kp}X_p\right)}{1 + \sum_{j=1}^{K-1} exp\left(\beta_{j0} + \beta_{j1}X_1 + \beta_{j2}X_2 + \cdots + \beta_{jp}X_p\right)} \text{ , } k < K \tag{2}$$

Where:

- $P(Y_i = k)$: The probability that the outcome Y_i falls into factor k.
- $(\beta_{k0}, \beta_{k1}, \dots, \beta_{kp})$: Coefficients for the predictors associated with factor k.
- $(X_1, X_2, \dots, X_p)$: The predictor variables used in the model.

Multinomial logistic regression is particularly well-suited for classification problems involving more than two categories. It is commonly used as an output node in neural network algorithms for tasks such as audio and text classification, where multiple categories are often present. However, in this study, the model is applied independently of neural networks to preserve the interpretability of the predictors, a crucial consideration for financial decision-making.

In multinomial logistic regression, one category is selected as the reference point. In this case, the Momentum factor is chosen as the reference category because it appears most frequently in the target variable, as illustrated in Table 1.

Table 1

Frequency of Factors in the Target Variable

Momentum	Quality	Value	Low Vol	Low Size
81	45	34	42	43

The model provides two types of outputs. First, the predicted class can be used to assign a higher weighting to the corresponding asset compared to others. Alternatively, the model's predicted probabilities for each asset can be utilized during rebalancing. By ranking the assets according to these probabilities and assigning weights based on their hierarchy, a more flexible, hierarchical asset allocation strategy is achieved. This approach allows for finer adjustments in portfolio composition, depending on the confidence of the model's predictions. Both allocation methods are shown in Figure 1.

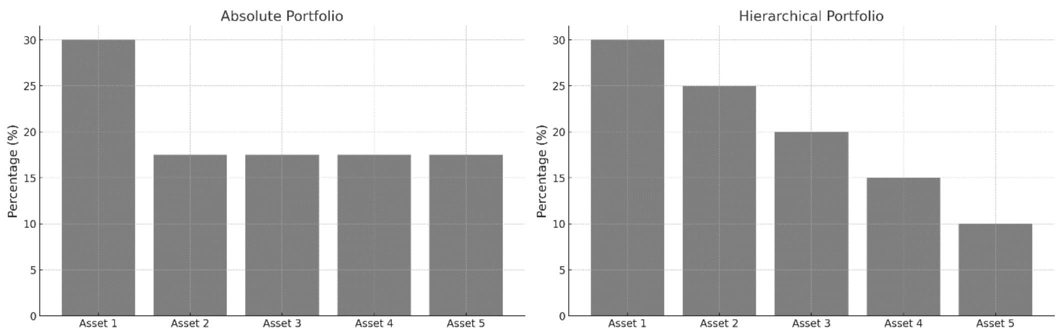

Figure 1: Example of Absolute and Hierarchical portfolio allocation

Regardless of the method chosen, constraints will be imposed on the portfolio weights to ensure diversification and risk management. Specifically, no asset will be allowed to exceed a 30% weight or fall below a 10% weight at the start of each rebalancing period.

To benchmark the performance of the proposed model, two additional portfolios with static weights will be constructed. The first is an equal-weighted portfolio, which allocates 20% to each asset at the beginning of the period. The second portfolio is optimized to maximize the Sharpe ratio, subject to the same weight constraints, as represented in equation (3).

75% of the sample data will be used to derive the Sharpe-optimized portfolio configuration and for model training and evaluation. The remaining 25%, covering the period from June 2019 to June 2024, will serve as an out-of-sample test set to evaluate the performance of all portfolios. Key performance metrics, including annualized return, annualized volatility, Sharpe ratio, and maximum drawdown, will be compared across the different portfolio strategies. The portfolios will be rebalanced at the beginning of each month, following the acquisition of data for the relevant exogenous variables. This systematic rebalancing approach allows for a consistent comparison across different time periods and market conditions.

4. Results

The multinomial logistic regression model provides crucial insights into how various macroeconomic and financial indicators influence the likelihood of each factor outperforming the Momentum factor, which serves as the reference category in this analysis. The coefficients for Low Size, Low Volatility, Quality, and Value indicate the impact of each predictor relative to Momentum, helping us understand the dynamics of factor performance under different economic conditions. Out of an initial pool of 14 variables, 8 were identified as the optimal configuration, as displayed in Table 3. The variables chosen include: NFCI, RSI, VIX, New Order, ShortM Yield, Break-even Infl., 1M Return Size and Spread Yield.

The NFCI (National Financial Conditions Index) measures the overall stress in the financial system, with higher values indicating tighter financial conditions. The results indicate that tighter financial conditions (a higher NFCI) are associated with a significant decrease in the likelihood of the Low Size and Low Volatility factors outperforming Momentum. This is evidenced by the negative coefficients for these factors, with the strongest impact observed for Low Size (-2.026, $p < 0.01$) and Low Volatility (-1.534, $p < 0.05$). This proposes that as financial conditions worsen, investors may shift away from smaller and less volatile stocks in favor of more resilient factors like Momentum. The Quality factor also exhibits a negative coefficient (-1.027), but it is not statistically significant, suggesting a weaker or less consistent relationship with NFCI. The Value factor shows an even weaker negative relationship, with a coefficient of -0.019, indicating its performance is largely unaffected by changes in financial conditions.

Spread Yield, which measures the risk premium between high-yield bonds and safer assets, has a significant positive association with the Low Size factor (3.511, $p < 0.01$). This indicates that during periods of increased market risk (wider spreads), smaller companies are more likely to outperform Momentum. The other factors, including Low Volatility, Quality, and Value, do not present significant relationships with the Spread Yield, advocating that their relative performance is less sensitive to changes in market risk as measured by this variable.

The Volatility Index (VIX) has a strong positive association with the Low Volatility factor (2.269, $p < 0.01$) and the Quality factor (1.124, $p < 0.05$). This indicates that during periods of heightened market volatility, investors are more likely to favor low-volatility and high-quality stocks, likely due to their perceived safety during turbulent times. On the other hand, the VIX negatively impacts the Value factor (-1.404, $p < 0.05$), suggesting that higher market fear reduces the attractiveness of value stocks relative to momentum strategies.

The New Orders for Durable Goods variable, representing economic activity, shows a negative association with Low Size (-0.625, $p < 0.1$) and Low Volatility (-0.686, $p < 0.05$) factors. This implies that an increase in new orders, indicative of economic expansion, decreases the likelihood of these factors outperforming Momentum. This may be because during economic growth, investors prefer momentum-driven strategies over the stability offered by low size and low volatility stocks. The Quality and Value factors do not exhibit a significant relationship with new orders.

Short-Term Yield captures the influence of short-term interest rates on factor performance and shows that rising short-term yields positively affect the Low Size factor (1.287, $p < 0.05$). This indicates that smaller companies are more likely to outperform Momentum when short-term interest rates increase, possibly due to their higher growth potential in a rising rate environment. The impact on the other factors is not significant, indicating that their relative performance is less sensitive to changes in short-term yields.

The Relative Strength Index (RSI), which measures market momentum, reveals that higher values, indicating overbought conditions, are positively associated with the likelihood of Low Size (0.822, $p < 0.05$) and Quality (0.938, $p < 0.05$) factors outperforming Momentum. This suggests that when the market is considered overbought, investors may favor smaller stocks and those of higher quality over momentum-driven strategies. Conversely, the Value factor has a negative but not statistically significant coefficient (-0.576), implying a weaker relationship with market momentum as captured by RSI.

Break-even Inflation Rate, an indicator of inflation expectations, exhibits a significant negative relationship with the Quality factor (-0.905, $p < 0.05$), implying that higher inflation expectations reduce the likelihood of quality stocks outperforming Momentum. The other factors do not show significant relationships with this variable, indicating a weaker connection to inflation expectations.

Finally, the 1M Return Size variable, representing the return of the size factor relative to the market, has a statistically significant negative coefficient for the Low Size factor (-0.995, $p < 0.05$), indicating that as smaller stocks perform better relative to the broader market, the likelihood of the Low Size factor outperforming the Momentum factor decreases. This suggests that strong relative performance by smaller stocks may reduce the chances of continued outperformance by the Low Size factor in the subsequent period. For the other factors (Low Volatility, Quality, and Value), the non-significant coefficients indicate that the size factor's relative performance does not significantly influence their likelihood of outperforming Momentum, with the Value factor showing a weak, positive but statistically insignificant relationship (0.474).

The results of the out-of-sample portfolios demonstrate the practical effectiveness of the multinomial logistic regression model in factor timing. As shown in Table 2, the Hierarchical and Absolute Portfolios, which are derived from the model, outperform the static Equal Weight and Sharpe-optimized portfolios in several key metrics.

Table 2

Portfolio out-of-sample results

	Hierarchical Portfolio	Absolute Portfolio	Equal Weight Portfolio	Sharpe Portfolio	Best Portfolio
Annual Return	**13.51%**	12.83%	12.21%	12.20%	16.22%
Volatility	18.28%	18.24%	18.35%	**17.92%**	18.38%
Max DD	-23.6%	-24.3%	-24.5%	**-23.3%**	-23.9%
Sharpe	**0.74**	0.70	0.67	0.68	0.88
Total Return	**88.44%**	82.86%	77.86%	77.80%	112.05%

The Hierarchical Portfolio achieves an annual return of 13.51%, outperforming both the Absolute Portfolio (12.53%) and the static portfolios. While these returns do not match the 16.22% annual return of the Best Portfolio, which represents the theoretical maximum achievable return by always overweighting the best-performing factor, the model-based portfolios still demonstrate a clear advantage over static approaches. This suggests that the model's ability to dynamically adjust factor weights based on predicted probabilities adds significant value.

In terms of risk, the Hierarchical Portfolio exhibits a volatility of 18.28%, which is slightly higher than the Absolute Portfolio (18.24%) and comparable to the static portfolios. However, the models still exhibit higher volatility than the Sharpe-optimized portfolio. The Max Drawdown (Max DD) for the Hierarchical Portfolio is -23.6%, which is marginally better than the Absolute Portfolio (-24.3%), better than the Equal Weight portfolio (-24.5%) and clearly comparable to the Sharpe portfolio (-23.3%).

The Sharpe Ratio for the Hierarchical Portfolio is 0.74, which is higher than the Absolute Portfolio (0.70), Equal Weight Portfolio (0.67), and Sharpe Portfolio (0.68), although it still falls short of the Best Portfolio's 0.88. This higher Sharpe Ratio indicates that the Hierarchical Portfolio offers superior risk-adjusted returns compared to both the Absolute Portfolio and the static portfolios.

The results highlight the value of using machine learning models for factor timing. Although the model-based portfolios do not achieve the theoretical maximum performance of the Best Portfolio, they outperform static allocation methods in terms of both return and risk-adjusted return. This demonstrates the practical utility of the model, not only in enhancing portfolio performance through dynamic factor allocation, but to understand the impact of economic cycles on our portfolio. However, the gap between the model-based portfolios and the Best Portfolio suggests that there is still room for improvement, potentially through further refinement of the model or the incorporation of additional predictors. This highlights the ongoing potential for machine learning models to evolve and deliver even greater value in the realm of portfolio management.

5. Conclusion

This study has delved into the application of machine learning techniques, specifically a multinomial logistic regression model, for factor timing in dynamic asset allocation. By incorporating a diverse set of macroeconomic, sentiment, momentum, and valuation indicators, the model has shown its potential to dynamically adjust portfolio allocations in response to changing market conditions. The results indicate that this approach can offer a significant advantage over traditional static allocation methods, as evidenced by the superior performance of the Hierarchical and Absolute Portfolios compared to the Equal Weight and Sharpe-optimized portfolios. The empirical results from the out-of-sample testing highlighted the practical utility of the model in enhancing portfolio performance. The Hierarchical Portfolio achieved a higher Sharpe Ratio and better risk-adjusted returns than both the Absolute Portfolio and the static allocation methods, underscoring the value of a machine learning-based approach to factor timing. However, despite these encouraging outcomes, the model-based portfolios did not reach the theoretical maximum performance of the Best Portfolio, which serves as an idealized benchmark.

In developing this model, potential gains in model accuracy were consciously sacrificed to maintain the interpretability of the predictors, thereby avoiding the "black box" nature of more complex algorithms like neural networks. This trade-off ensures that the model remains transparent and understandable, which is crucial for practical decision-making in finance.

For further steps, future research should focus on enhancing the robustness of the model results by performing cross-validations. This would ensure that the findings are not only accurate but also generalizable across different market conditions. Additionally, future studies could benefit from using proprietary factor indices and extending the time range of the data to provide a more comprehensive analysis. Expanding the dataset and refining the model could potentially close the gap between the model-derived portfolios and the Best Portfolio, paving the way for more sophisticated and precise investment strategies.

In conclusion, this paper demonstrates the transformative potential of machine learning in asset allocation and factor timing. While the results are promising, they also suggest that the field is ripe for further innovation. As machine learning models continue to evolve, there is significant potential for these tools to redefine the standards of portfolio management, offering more adaptive, resilient, and optimized investment strategies in an increasingly complex financial landscape.

6. Appendices

Methodology

Normalization

Normalization is a crucial preprocessing step in the development of any statistical or machine learning model, including multinomial logistic regression. The process of normalization involves rescaling the values of variables so that they have a mean of 0 and a variance of 1. This is particularly important when dealing with variables that have different units or scales, as it ensures that no single variable disproportionately influences the model due to its larger scale.

In the context of multinomial logistic regression, normalization is essential for several reasons. Firstly, it facilitates the convergence of the model during the optimization process. Logistic regression relies on iterative methods to find the optimal set of parameters, and when the variables are on different scales, the algorithm might struggle to converge efficiently. This can lead to longer computation times or even failure to converge, resulting in an unreliable model.

Secondly, normalization improves the interpretability of the model coefficients. When the variables are normalized, the coefficients represent the change in the log-odds of the dependent variable per one standard deviation change in the predictor. This standardization allows for a more meaningful comparison between the effects of different variables, making it easier to identify which predictors have the most significant impact on the outcome.

Furthermore, normalization is particularly beneficial in our multinomial logistic regression model because it helps to address issues related to multicollinearity. When variables are on vastly different scales, multicollinearity, where predictors are highly correlated, can become more pronounced, leading to inflated standard errors and unreliable coefficient estimates. By normalizing the variables, we reduce the risk of such issues, thus enhancing the stability and reliability of our model.

In addition to standard normalization, some variables in our dataset required a logarithmic transformation due to their asymmetric nature. Logarithmic transformation is applied to variables that exhibit a skewed distribution, where most of the data points are clustered at one end of the scale. This transformation helps to stabilize the variance and make the data more symmetric, which is beneficial for the regression model as it assumes that the residuals (differences between observed and predicted values) are normally distributed. By applying a logarithmic transformation, we ensure that the variables meet this assumption, further improving the accuracy and reliability of the model.

Variables

Figure 2 presents the time series of six key macroeconomic variables used in the analysis, spanning from 2004 to 2024. The variables include the US High Yield Index Option-Adjusted Spread, National Financial Conditions Index, 10-Year Breakeven Inflation Rate, Manufacturers' New Orders, Yield on U.S. Treasury Securities at 3-Month Maturity, and the Consumer Price Index. Each time series captures the fluctuations in these economic indicators, reflecting significant economic events such as the 2008 financial crisis and the COVID-19 pandemic, and their impacts on market conditions over time.

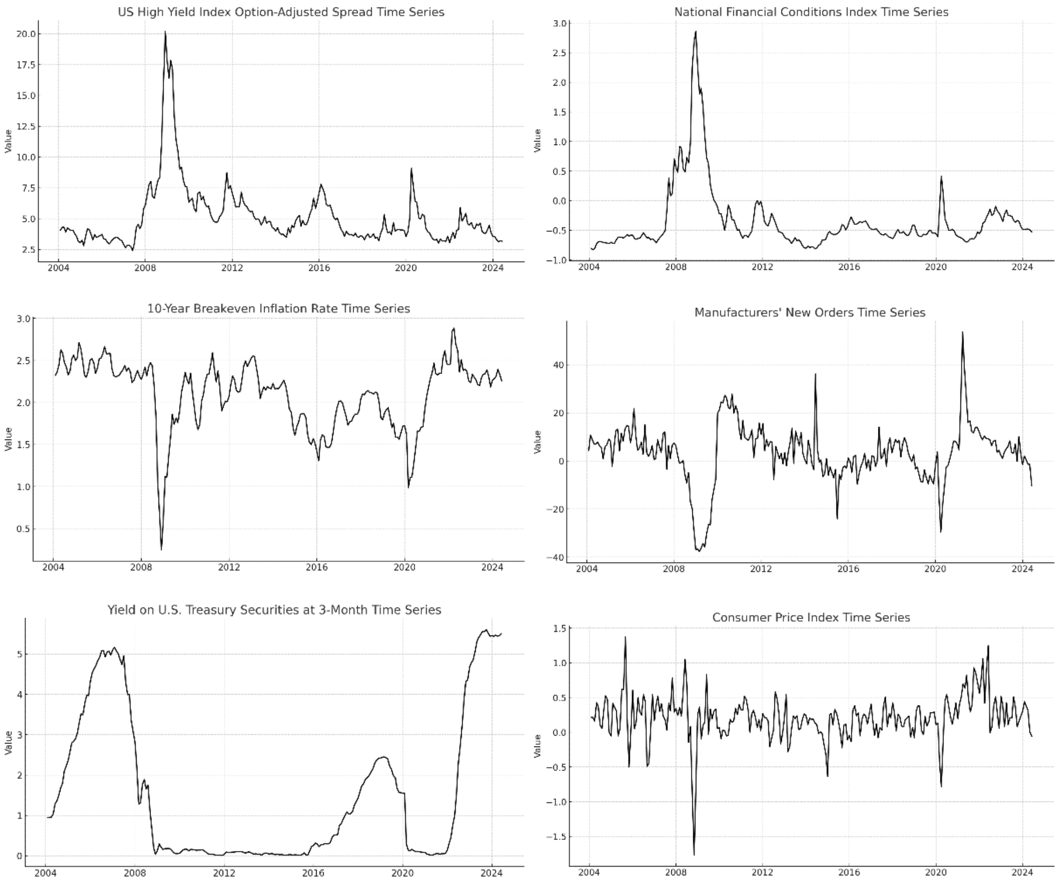

Figure 2: Time series of macro variables

Figure 3 displays the time series of momentum-related variables used in the study. The top graph shows the cumulative returns of five investment factors: Momentum, Quality, Low Volatility, Value, and Low Size, from 2004 to 2024, highlighting the relative performance of each factor over time. The bottom two graphs illustrate additional momentum indicators: the S&P 500 Above SMA(200) Ratio, which tracks the percentage of stocks trading above their 200-day moving average, and the S&P 500 RSI Mean, which represents the average Relative Strength Index of S&P 500 stocks.

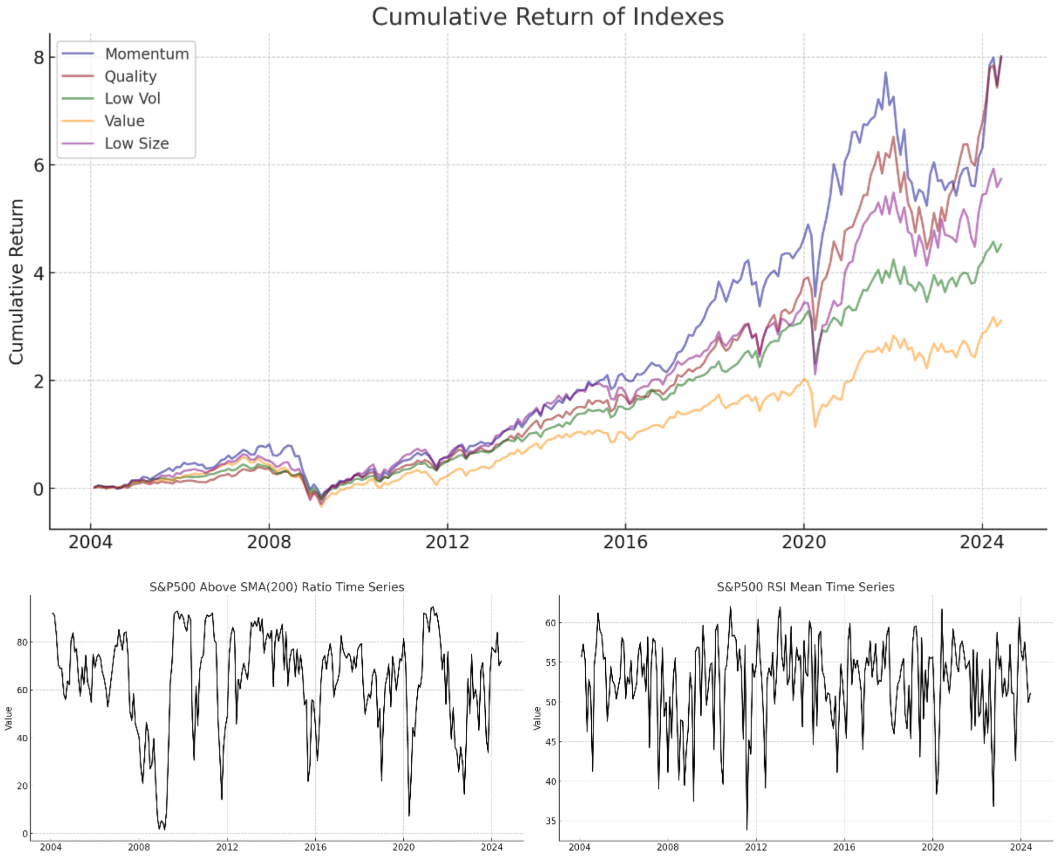

Figure 3: Time series of momentum variables

Figure 4 shows the time series of sentiment variables used in the study. The left graph in Figure 4 shows the Short Interest Ratio Time Series, which tracks the average short interest ratio of stocks over time, indicating market sentiment and investor pessimism. The right graph in Figure 4 displays the Volatility Index (VIX) Time Series which measures market volatility expectations. Figure 5 displays the time series of value variables, specifically the Risk Premium and Long-Term Earnings Growth. The Risk Premium captures the additional return investors demand for taking on more risk, while Long Term Earnings Growth represents expected future earnings growth of the market, based on the Gordon Growth Model.

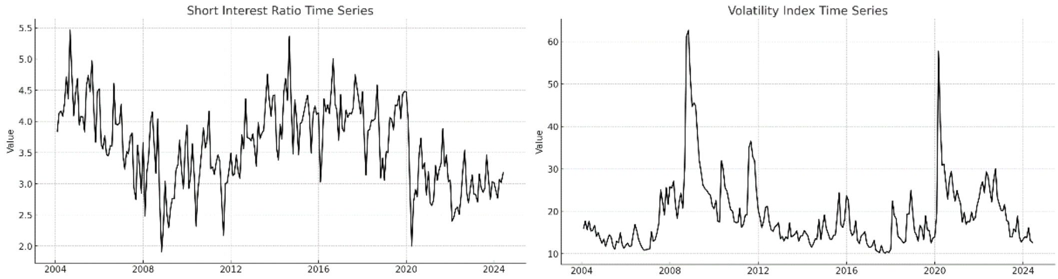

Figure 4: Time series of sentiment variables

Figure 5:Time series of value variables

The provided heatmap in Figure 6 illustrates the correlation matrix of non-normalized variables used in the study, capturing the strength and direction of linear relationships between pairs of variables. The color scale ranges from deep blue, indicating strong negative correlations, to deep red, indicating strong positive correlations, with neutral colors representing weak or no correlations.

The heatmap reveals several noteworthy patterns. Among the factor returns, there are strong positive correlations, particularly between Ret_Value and Ret_Quality, which share a correlation of 0.96, indicating that these two factors tend to move closely together. This suggests that when the value factor performs well, the quality factor is likely to perform similarly. The other factor returns (Ret_Momentum, Ret_Low_Vol, and Ret_Size) also exhibit high positive correlations with each other, reflecting their interconnected behavior in the market.

Sentiment indicators like the Volatility Index (VIX) show strong positive correlations with Spread_Yield (0.88) and NFCI (0.78). This implies that periods of high market volatility are typically associated with increased risk premiums and tighter financial conditions, as reflected by the higher spread yields and NFCI values. These relationships suggest that volatility is a key driver of risk perception in financial markets.

Macroeconomic indicators are also interconnected, with New_Order showing a moderate negative correlation with NFCI (-0.54), suggesting that increases in new orders for durable goods are associated with easing financial conditions. Additionally, the negative correlation between ShortM_Yield and NFCI (-0.54) aligns with the expectation that higher short-term yields coincide with tighter financial conditions, reinforcing the relationship between monetary policy and market stress.

Valuation indicators such as LT_EarningsGr exhibit a moderate positive correlation with Risk Premium (0.60), indicating that periods of higher long-term earnings growth are typically associated with higher risk premiums. Conversely, LT_EarningsGr has a moderate negative correlation with Spread_Yield (-0.58), suggesting that higher earnings growth is often accompanied by lower spreads between high-yield bonds and safer assets, reflecting investor confidence.

The heatmap also highlights cross-category relationships. For instance, RSI, a momentum indicator, shows a positive correlation with Ret_Momentum (0.67), reinforcing the intuitive link between momentum indicators and the actual returns of momentum-based strategies.

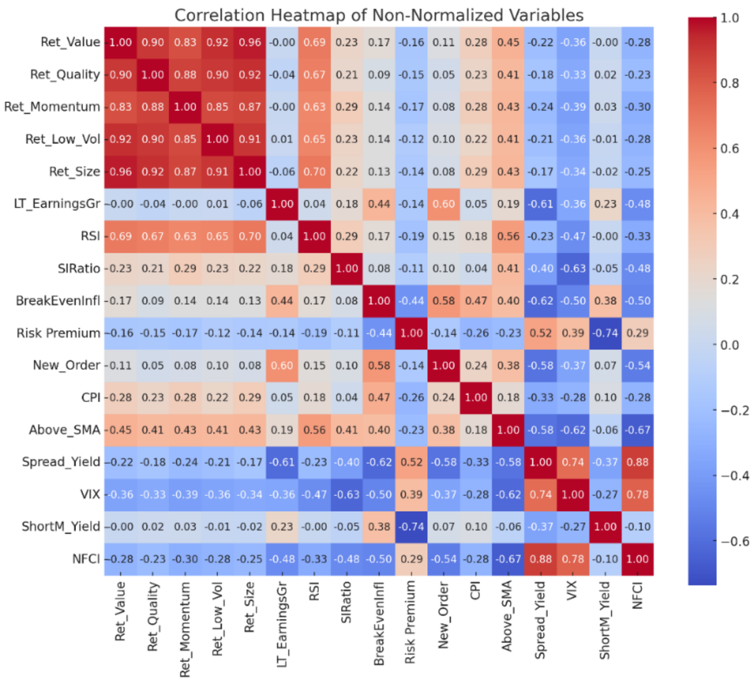

Figure 6: Correlation Heatmap of Non-Normalized Variables

Model and Portfolio

Figure 7 compares two portfolio strategies: the Sharpe Portfolio (left) and the Equal Weight Portfolio (right). The Sharpe Portfolio is optimized for maximum risk-adjusted returns, resulting in varied allocations, with higher weights in Momentum and Low Volatility. In contrast, the Equal Weight Portfolio allocates an equal 20% to each factor, offering a simple, balanced exposure without optimization.

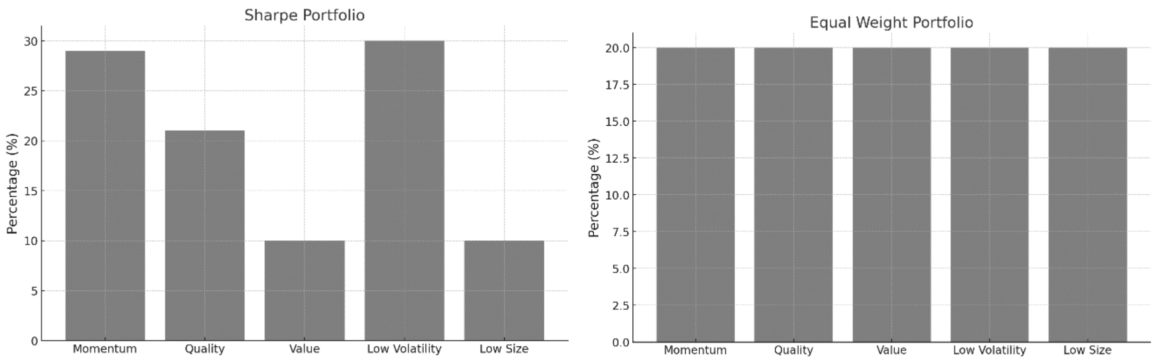

Figure 7: Example of static Equal Weight and Sharpe portfolio allocation

Equation (3) represents the mathematical formulation of the Sharpe ratio optimization problem used to determine the optimal portfolio weights under special constraints. The objective is to maximize the Sharpe ratio, which is the ratio of the expected portfolio return to its standard deviation, thereby maximizing risk-adjusted return.

$$\max_w \left(\frac{\sum_{i=1}^{N} w_i \, E(R_i)}{\sqrt{w^T \Sigma w}}\right) \tag{3}$$

subject to:

$$w^T 1 = 1 \tag{4}$$

$$0.1 \leq w_i \leq 0.3 \tag{5}$$

Where:

- w_i is the portfolio weight of asset *i.*
- $E(R_i)$ is the expected return of asset *i.*
- Σ is the covariance matrix of the asset returns.
- 1 is a vector of ones.

Results

Table 3

Multinomial logistic regression analysis

	Dependent Variable			
	Low Size	**Low Vol**	**Quality**	**Value**
	(1)	(2)	(3)	(4)
NFCI	-2.026***	-1.534**	-1.027	-0.019
	(0.733)	(0.745)	(0.669)	(0.693)
Spread Yield	3.511***	-0.585	0.249	1.476
	(0.964)	(0.839)	(0.789)	(0.922)
RSI	0.822**	0.338	0.938**	-0.576
	(0.412)	(0.375)	(0.403)	(0.445)
1M Return Size	-0.995**	-0.300	-0.420	0.474
	(0.412)	(0.399)	(0.377)	(0.508)
Break-even Infl.	0.061	-0.245	-0.905**	0.080
	(0.353)	(0.393)	(0.377)	(0.407)
VIX	-0.687	2.269***	1.124**	-1.404**
	(0.649)	(0.573)	(0.530)	(0.691)
New Order	-0.625*	-0.686**	-0.233	0.470
	(0.337)	(0.342)	(0.326)	(0.363)
ShortM Yield	1.287**	-0.199	0.691	0.309
	(0.510)	(0.534)	(0.496)	(0.524)
Constant	-1.414***	-0.659**	-0.661**	-1.526***
	(0.390)	(0.269)	(0.263)	(0.408)
Akaike Inf. Crit.	548.855	548.855	548.855	548.855
Note:		***p<0.01;	**p<0.05;	*p<0.1

Figure 8 illustrates the cumulative returns of four distinct portfolio strategies over the out-of-sample testing period. The Hierarchical Portfolio, represented by the blue line, exhibits the highest cumulative return at the conclusion of the period, indicating its superior performance relative to the other strategies. While the portfolios exhibit similar trajectories in the early stages, the Hierarchical Portfolio progressively outperforms, particularly in the later stages, underscoring the efficacy of dynamic factor timing in enhancing portfolio returns.

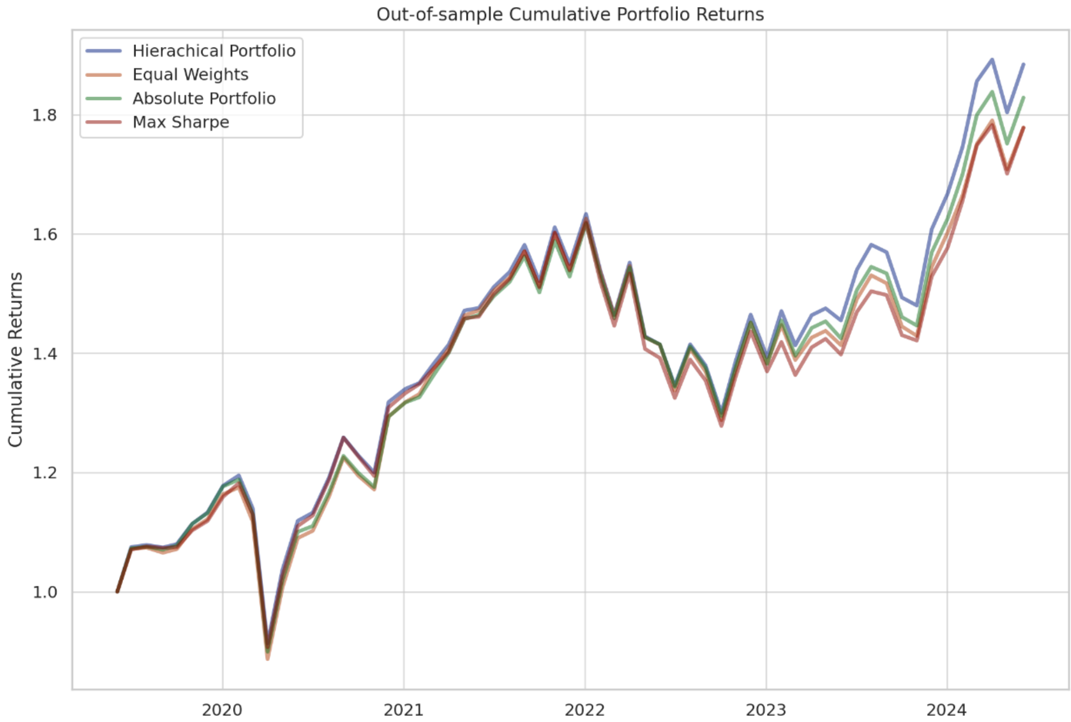

Figure 8: Out-of-sample Portfolio cumulative returns

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